BOSWELL-BÈTA

# James Boswell Exam
# VWO Mathematics C

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|---|---|
| Date: | Practice exam 1 |
| Time: | 3 hours |
| Number of questions: | 6 |
| Number of subquestions: | 20 |
| Number of supplements: | 1 |
| Total score: | 68 |

- Write your name **on every sheet** of paper you hand in.
- Use a separate sheet of paper for each question.
- For each question, show how you obtained your answer either by means of a calculation or, if you used a graphing calculator, an explanation. Otherwise, no points will be awarded to your answer.
- Make sure that your handwriting is legible and write in blue or black non-erasable ink. No correction fluid of any kind is permitted. Use a pencil only to draw graphs and geometric figures.
- You may use the following:
    - Graphing calculator;
    - Drawing utensils;
    - List of formulas;
    - Dictionary, subject to the approval of the invigilator.

# Question 1: Electric cars

Chris wants to place an order for two electric cars of the same type: one for himself and one for his son.

You have several options when ordering this car:

- For the paint colour you can choose between white, black, grey, blue or red.
- For the rims, you can choose between rims of 19 inch or rims of 21 inch.
- For the colour of the interior you can choose between white, black or cream-coloured.

The two cars that Chris orders do not have to be the same.

3p **a** Calculate how many different orders Chris can place.

The mechanical power output of a car engine depends on the speed of the car. For the cars that Chris wants to buy, the following formula holds:

$$P = 0.00002 \cdot v^3 - 0.0025 \cdot v^2 + 0.24 \cdot v$$

Here $P$ is the mechanical power output in kilowatts and $v$ is the speed in km per hour.
The formula is only valid for $20 \leq v \leq 140$.

In figure 1 the graph of $P$ has been drawn.

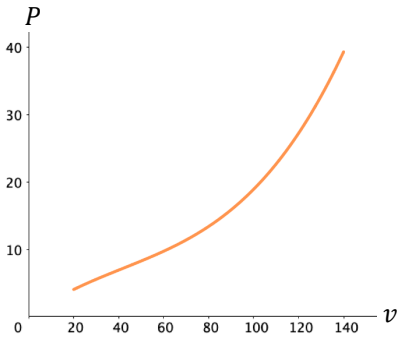



**figure 1**

3p **b** Calculate by what percentage the mechanical power output increases when the speed increases from 100 km per hour to 130 km per hour. Round your answer to one decimal place.

The mechanical power output of a car engine can also be expressed in horsepower (hp). 1 kilowatt is equal to 1,36 hp.

4p **c** Calculate the speed of Chris' car when the mechanical power output is 20 hp. Round your answer to one decimal place.

The energy consumption of the cars that Chris wants to buy can be calculated with the formula:

$$E = \frac{100 \cdot P}{v}$$

Here $E$ is the energy consumption in kilowatt-hours per 100 km, $P$ is the mechanical power output in kilowatts and $v$ is the speed in km per hour.

In figure 2 the graph of $E$ has been drawn. At a certain speed, the energy consumption is minimal.

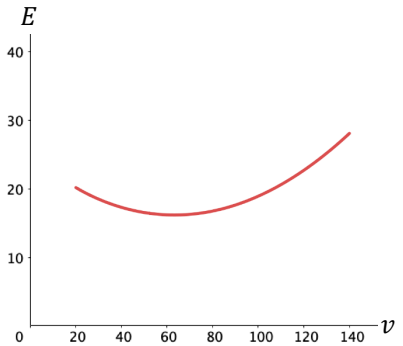
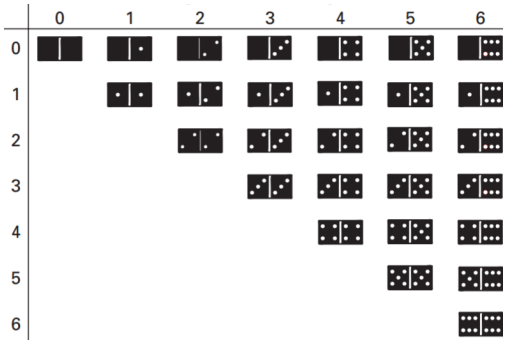



**figure 2**

3p **d** Calculate the minimal energy consumption.

# Question 2: Dominos

The game of dominos is played with $28$ domino pieces. Every domino consists of two halves, each of which contains either $0, 1, 2, 3, 4, 5$ or $6$ dots. In the figure below you see all $28$ different dominos. The figure is enlarged on the supplement to this exam. There are seven so-called **doubles** among the dominos: those are the dominos with the same number of dots on both halves.




Kim and Lucas play dominos together. At the beginning of the game they put all $28$ dominos upside down on the table. Then they take turns to draw seven dominos blindly. Kim is the first to take seven dominoes.

3p **a** Calculate the probability that Kim does not draw any double. Round your answer to three decimal places.

Kim looks at the dominos she has drawn. It turns out that she drew one double: the domino with $5$ dots on both halves. According to the rules of the game, the player with 'highest' double may start the game. This means that Lucas may only start the game if he draws the domino with $6$ dots on both halves.

3p **b** Calculate the probability that Lucas may start the game. Give your answer as a fraction or round to three decimal places.

After Kim and Lucas both have their dominos, the game starts. From the dominos still on the table, one is taken and turned upwards. The person who starts the game has to put a 'connecting' domino next to it. 'Connecting' means that the dominos have the same number of dots on their adjacent halves. See figure $2$.

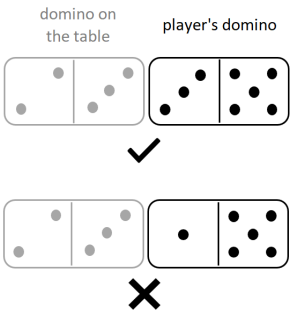
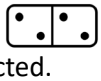
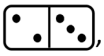



**figure 2**

Lucas poses the following question: if all $28$ dominos are put on the table and I take two of them blindly, what is the probability that those two dominos can be connected?

Kim explains that it makes a difference whether the first drawn domino is a double or not. An example:

- If you first draw  , only dominos with a $2$ can be connected.
- If you first draw  , all dominos with a $2$ or a $3$ can be connected.

5p **c** Answer Lucas' question. You can use the figure on the supplement to this exam.

## Question 3: A sequence

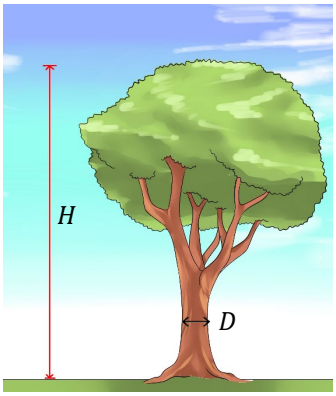
For a certain sequence $u$ the following holds: $u_2 = 10\,000$ and $u_6 = 4096$.
The first term of the sequence $u$ has rank (index) $0$.

3p **a** Suppose that $u$ is an arithmetic sequence (linear sequence).
Give a recursive formula for the sequence $u$.

3p **b** Suppose that $u$ is not an arithmetic, but a geometric sequence (exponential sequence).
Calculate the first term $u_0$.

# Question 4: Trees

In an American study, the height $H$ and the diameter $D$ (at one metre above the ground) of a large number of trees have been measured, both in metres.




The following table contains two measurements from the study:

| $H$ | $D$ |
|---|---|
| 3.15 | 0.06 |
| 12.62 | 0.45 |

3p **a** Show that $H$ and $D$ are neither directly proportional nor inversely proportional.

The study found the following formula for the relationship between $D$ and $H$:

$$D = 0.01 \cdot H^{1.5} \qquad \text{(with } D \text{ and } H \text{ both in metres)}$$

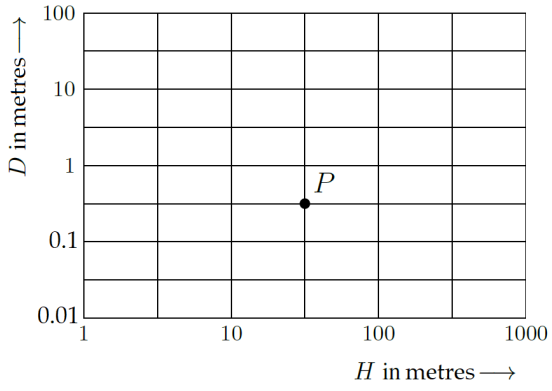
The giant sequoia is one of the largest trees in the world. The diameter of the tree (at one metre above the ground) is 7.7 metres on average.

3p **b** According to the formula, what is the height of a giant sequoia? Round your answer to the nearest metre.

You can write the formula $D = 0.01 \cdot H^{1.5}$ in the form $H = p \cdot D^q$.

4p **c** Show this and give the values of $p$ and $q$ accurate to two decimal places.

The researchers plotted the measurements in a coordinate system with a logarithmic scale on both axes. This coordinate system is shown in the figure below. The figure is enlarged on the supplement to this exam.




Point $P$ belongs to one of the measurements from the study.

3p **d** Determine the diameter and the height of the tree corresponding to point $P$ using the figure on the supplement (so without using the formula). Round your answers to one decimal place. Show your workings.

# Question 5: Three teachers

An educational institute offers courses in mathematics, physics and chemistry.

The following is known about the teachers at this institute:

- The maths teachers always tell the truth.
- The physics teachers never tell the truth.
- The chemistry teachers sometimes tell the truth, but they also sometimes lie.

This question is about three teachers: a maths teacher, a physics teacher and a chemistry teacher.

The teachers make the following statements:

- Alice: 'Carlos is a maths teacher.'
- Bob: 'That is not true. Carlos is a physics teacher.'
- Carlos: 'No I'm not. I am a chemistry teacher.'

3p **a** Explain why Carlos cannot be a maths teacher.

4p **b** Determine which course Alice, Bob and Carlos each teach. Explain your answer clearly.

# Question 6: Coffee

Coffee contains caffeine, a substance that makes you more alert. After drinking coffee, the caffeine enters the bloodstream. Subsequently, the caffeine in the blood is broken down by the human body. The amount of caffeine in the blood decreases exponentially.

One hour after drinking a cup of coffee, the initial amount of caffeine has decreased by $12\%$.

3p **a** Calculate how long it takes for the human body to break down half the initial amount of caffeine. Give your answer in hours and round to one decimal place.

The body of a pregnant woman breaks down caffeine more slowly. Bibi is pregnant and drinks a cup of coffee in the morning. The amount of caffeine in her blood can be calculated with the following formula:

$$C = 40 \cdot 0.932^t$$

Here $C$ is the amount of caffeine (in mg) and $t$ is the time in hours, with $t = 0$ corresponding to $09{:}00$.

5p **b** Express $t$ in $C$ and use the formula thus obtained to calculate the time at which only $8$ mg of caffeine will be present in Bibi's blood. Write your answer in the form *hh:mm.*

Bibi makes her coffee with a coffee machine. She sets the machine to dispense $120$ mL of coffee per cup. Assume that the amount of coffee that the machine dispenses per cup is normally distributed with a mean of $120.0$ mL and a standard deviation of $3.5$ mL.

2p **c** Calculate how many mL of coffee the $25\%$ most filled cups at least contain. Round your answer to one decimal place.

Bibi has visitors and makes five cups of coffee using her machine.

4p **d** Calculate the probability that two of the five cups contain more than $125.0$ mL of coffee. Round your answer to three decimal places.

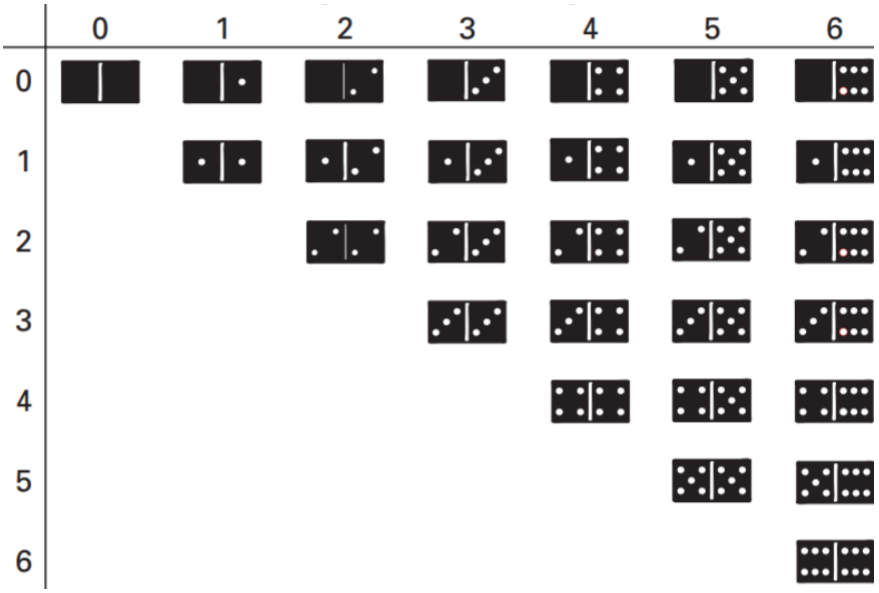
4p **e** Calculate the probability that the five cups together contain between $595.0$ mL and $605.0$ mL of coffee. Round your answer to three decimal places.

# Supplement

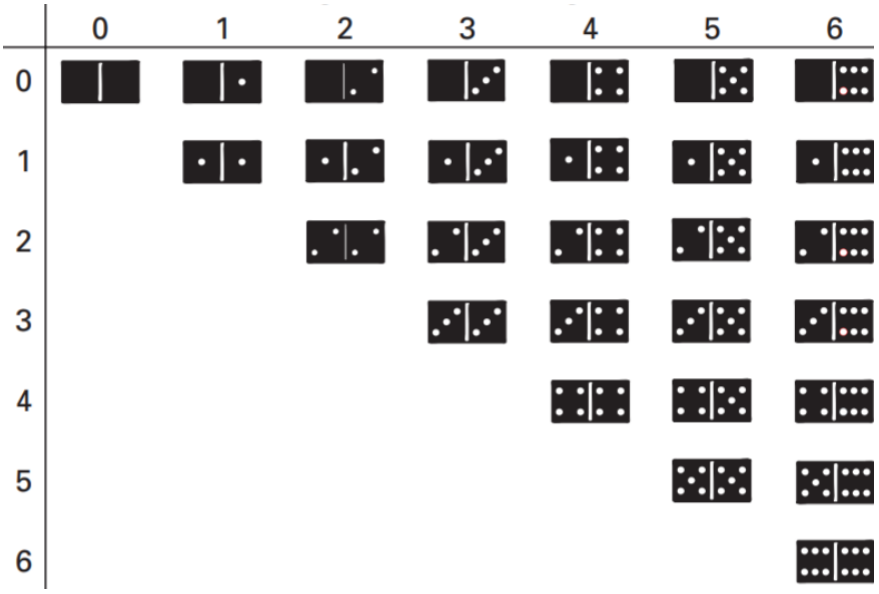
VWO Mathematics C Example Exam 1

Name: _____

**2**




**2c**



**5d**